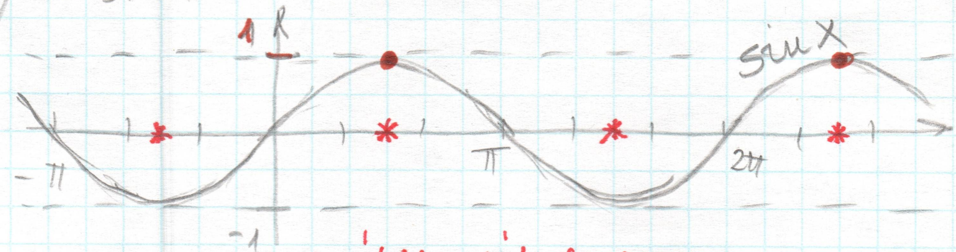
1. $\frac{1}{\cos x} = \operatorname{tg} x$

K.: $\cos x \neq 0$

$x \neq \frac{\pi}{2} + k \cdot \pi$

$\frac{1}{\cos x} = \frac{\sin x}{\cos x} \quad / \cdot \cos x \neq 0$

$1 = \sin x$



*: 'tiltott' helyek..

Ahol $\sin x = 1$, ott $\cos x = 0$

$\Rightarrow$ Nincs megoldás.

2. $\cos^4 x + \sin^4 x = \frac{3}{4}$

Emeljük négyzetre a

$\cos^2 x + \sin^2 x = 1$ -et!

(Mindkét old. poz.)

$\cos^4 x + 2 \cdot \sin^2 x \cos^2 x + \sin^4 x = 1$

$\cos^4 x + \sin^4 x = 1 - 2 \sin^2 x \cos^2 x$

Ezt az egyenletbe beírva:

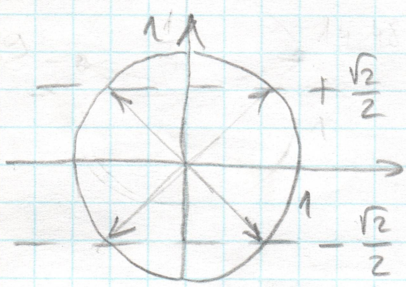
$1 - 2 \cdot \sin^2 x \cos^2 x = \frac{3}{4}$

$\frac{1}{4} = 2 \sin^2 x \cos^2 x \quad / \cdot 2$

$\frac{1}{2} = 4 \cdot \sin^2 x \cos^2 x$

$\frac{1}{2} = \sin^2 2x$

$\sin 2x = \frac{\sqrt{2}}{2}$ $\qquad$ $\sin 2x = -\frac{\sqrt{2}}{2}$

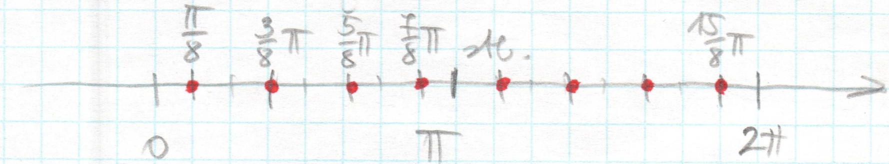


Összevonva: $\left(\sin 45° = \frac{\sqrt{2}}{2}\right)$

$2x = \frac{\pi}{4} + k \cdot \frac{\pi}{2} \quad / : 2$

$\cancel{\boxed{x = \frac{\pi}{8} + k \cdot \frac{\pi}{4} \; ; k \in \mathbb{Z}}}$

A $[0; 2\pi]$-n:



$x \in \left\{ \frac{\pi}{8} ; \frac{3}{8}\pi ; \frac{5}{8}\pi ; \frac{7}{8}\pi ; \frac{9}{8}\pi ; \frac{11}{8}\pi ; \frac{13}{8}x ; \frac{15}{8}\pi \right\}$

4. $\cos x + \sin x = \frac{1}{3}$

$x = 2y$

$\cos 2y + \sin 2y = \frac{1}{3}$

$\cos^2 y - \sin^2 y + 2\sin y \cdot \cos y = \frac{1}{3} \,/\cdot 3$

$3\cos^2 y - 3\sin^2 y + 6\sin y \cdot \cos y = 1$

$3\cos^2 y - 3\sin^2 y + 6\sin y \cos y = \sin^2 y + \cos^2 y \,/ \cos^2 y$

$F: \cos y \neq 0$

$3 - 3\operatorname{tg}^2 y + 6\operatorname{tg} y = \operatorname{tg}^2 y + 1$

$z = \operatorname{tg} y$

$3 - 3z^2 + 6z = z^2 + 1$

$0 = 4z^2 - 6z - 2$

$0 = 2z^2 - 3z - 1$

$z_1 = 1{,}781$

$\operatorname{tg} y_1 = 1{,}781$

$y_1 = 60{,}69^\circ + k \cdot 180^\circ$

$z_2 = -0{,}2808$

$\operatorname{tg} y_2 = -0{,}2808$

$y_2 = -15{,}68^\circ + k \cdot 180^\circ$

$y = \frac{x}{2}$

$\frac{x_1}{2} = 60{,}69^\circ + k \cdot 180^\circ$

$x_1 = 121{,}4^\circ + k \cdot 360^\circ$

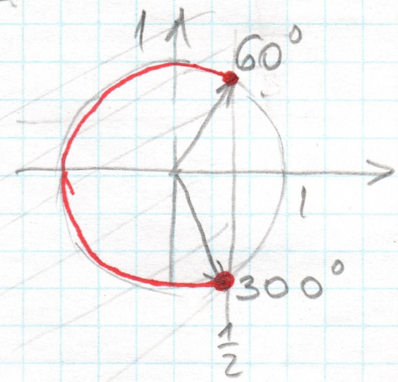
$\frac{x_2}{2} = -15{,}68 + k \cdot 180^\circ$

$x_2 = -31{,}37^\circ + k \cdot 360^\circ$

$$\boxed{x_1 = 2{,}118 + k \cdot 2\pi \,;\, k \in \mathbb{Z} \,;\, x_2 = -0{,}5475 + k \cdot 2\pi}$$

3. $\cos^4 x - \sin^4 x \le \frac{1}{2}$ $\qquad x \in [-360°; 360°]$

$$\underbrace{(\cos^2 x - \sin^2 x)}_{\cos 2x}\underbrace{(\cos^2 x + \sin^2 x)}_{1} \le \frac{1}{2}$$

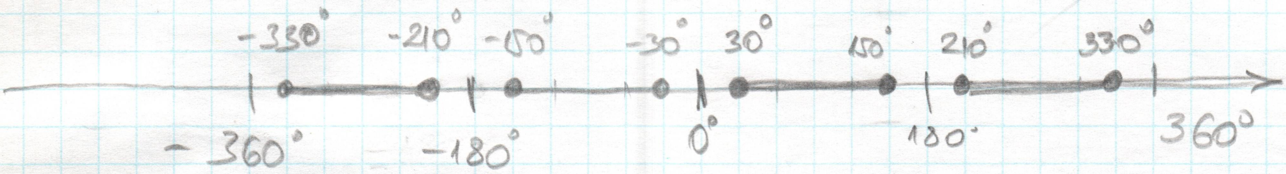


$$\cos 2x \le \frac{1}{2}$$

$$60° + k\cdot 360° \le 2x \le 300° + k \cdot 360°$$

$$30° + k \cdot 180° \le x \le 150° + k \cdot 180°$$

$[-360°; 360°]$-on:

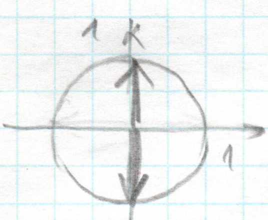


$$\boxed{M:\ x \in [-330°; -210°] \cup [-150°; -30°] \cup [30°; 150°] \cup [210°; 330°]}$$

5. $\dfrac{\cos 3x}{1 - \sin x} = 0 \quad / \cdot \underset{\neq 0}{(1 - \sin x)}$ $\qquad$ K.: $\sin x \neq 1$

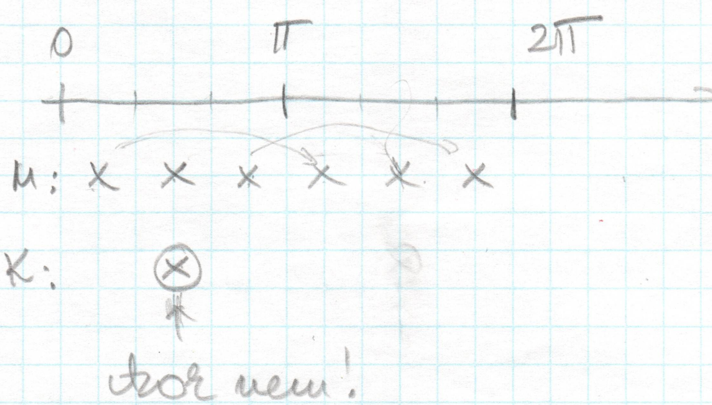
$\qquad x \neq \frac{\pi}{2} + k \cdot 2\pi$

$\cos 3x = 0$



$3x = \frac{\pi}{2} + k \cdot \pi$

$x = \frac{\pi}{6} + k\frac{\pi}{3}$



$$\boxed{\begin{aligned} x_1 &= \frac{\pi}{6} + k\cdot\pi \\ x_2 &= \frac{5}{6}\pi + k \cdot \pi \qquad k \in \mathbb{Z} \\ x_3 &= \frac{3}{2}\pi + k \cdot 2\pi \end{aligned}}$$

6. $(\sin x - 1)(\operatorname{tg} x - 1) \leq 0$

K.: ($\operatorname{tg} x$ miatt)

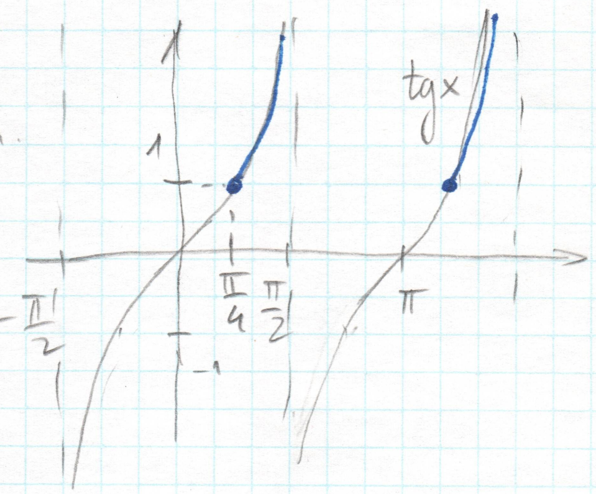
$x \neq \frac{\pi}{2} + k\cdot\pi$

Mivel $\sin x \leq 1$, ezért

$\sin x - 1 \leq 0$

$\Rightarrow$ $\operatorname{tg} x - 1 \geq 0$ kell legyen.

$\operatorname{tg} x \geq 1$



$$\boxed{M:\ x \in \left[\frac{\pi}{4} + k\cdot\pi ; \frac{\pi}{2} + k\cdot\pi\right[\quad k \in \mathbb{Z}}$$

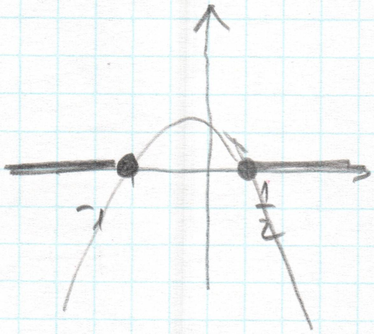
(7.) $2\sin^2 x - \cos x - 1 \le 0$

$A = \cos x$ $\qquad \sin^2 x = 1 - \cos^2 x = 1 - A^2$

$2(1 - A^2) - A - 1 \le 0$

$-2A^2 - A + 1 \le 0$

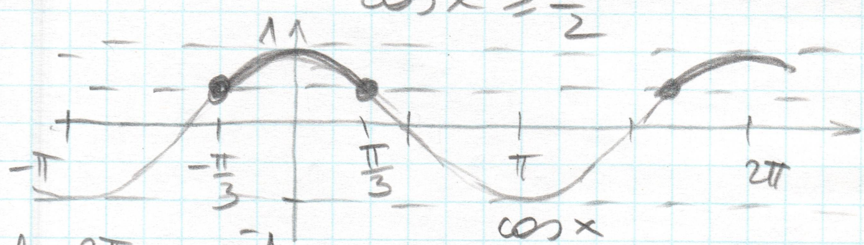
$A_1 = -1$ $\qquad A_2 = \frac{1}{2}$

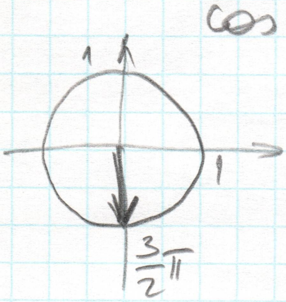


$A \le -1$ vagy $A \ge \frac{1}{2}$

$\cos x \le -1$ $\qquad \cos x \ge \frac{1}{2}$

Csak úgy lehet, ha $\cos x = -1$





$x_1 = \frac{3}{2}\pi + k \cdot 2\pi$

$x_2 \in \left[-\frac{\pi}{3} + k \cdot 2\pi \,;\, \frac{\pi}{3} + k \cdot 2\pi\right]$

$\boxed{\text{M.: } x \in \left[-\frac{\pi}{3} + k \cdot 2\pi \,;\, \frac{\pi}{3} + k \cdot 2\pi\right] \cup \left\{\frac{3}{2}\pi + k \cdot 2\pi\right\};\ k \in \mathbb{Z}}$